Ministry of Education

Ontario

The Ontario Curriculum
Grades 9 and 10

REVISED

Mathematics

2005

The Mathematical Processes

Presented at the start of every course in this curriculum document is a set of seven expectations that describe the mathematical processes students need to learn and apply as they work to achieve the expectations outlined within the strands of the course. In the 1999 mathematics curriculum, expectations relating to the mathematical processes were embedded within individual strands. The need to highlight these process expectations arose from the recognition that students should be actively engaged in applying these processes throughout the course, rather than in connection with particular strands.

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The mathematical processes are interconnected. Problem solving and communicating have strong links to all the other processes. A problem-solving approach encourages students to reason their way to a solution or a new understanding. As students engage in reasoning, teachers further encourage them to make conjectures and justify solutions, orally and in writing. The communication and reflection that occur during and after the process of problem solving help students not only to articulate and refine their thinking but also to see the problem they are solving from different perspectives. This opens the door to recognizing the range of strategies that can be used to arrive at a solution. By seeing how others solve a problem, students can begin to think about their own thinking (metacognition) and the thinking of others, and to consciously adjust their own strategies in order to make their solutions as efficient and accurate as possible.

The mathematical processes cannot be separated from the knowledge and skills that students acquire throughout the course. Students must problem solve, communicate, reason, reflect, and so on, as they develop the knowledge, the understanding of concepts, and the skills required in the course.

Problem Solving

Problem solving is central to learning mathematics. It forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction. It is considered an essential process through which students are able to achieve the expectations in mathematics, and is an integral part of the mathematics curriculum in Ontario, for the following reasons. Problem solving:

- is the primary focus and goal of mathematics in the real world;
- helps students become more confident mathematicians;
- allows students to use the knowledge they bring to school and helps them connect mathematics with situations outside the classroom;
- helps students develop mathematical understanding and gives meaning to skills and concepts in all strands;
- allows students to reason, communicate ideas, make connections, and apply knowledge and skills;
- offers excellent opportunities for assessing students' understanding of concepts, ability to solve problems, ability to apply concepts and procedures, and ability to communicate ideas;
- promotes the collaborative sharing of ideas and strategies, and promotes talking about mathematics;
- helps students find enjoyment in mathematics;
- increases opportunities for the use of critical-thinking skills (e.g., estimating, classifying, assuming, recognizing relationships, hypothesizing, offering opinions with reasons, evaluating results, and making judgements).

Not all mathematics instruction, however, can take place in a problem-solving context. Certain aspects of mathematics must be explicitly taught. Conventions, including the use of mathematical symbols and terms, are one such aspect, and they should be introduced to students as needed, to enable them to use the symbolic language of mathematics.

Selecting Problem-Solving Strategies. Problem-solving strategies are methods that can be used successfully to solve problems of various types. Teachers who use relevant and meaningful problem-solving experiences as the focus of their mathematics class help students to develop and extend a repertoire of strategies and methods that they can apply when solving various kinds of problems – instructional problems, routine problems, and non-routine problems. Students develop this repertoire over time, as they become more mature in their problem-solving skills. By secondary school, students will have learned many problem-solving strategies that they can flexibly use and integrate when faced with new problem-solving situations, or to learn or reinforce mathematical concepts. Common problem-solving strategies include the following: making a model, picture, or diagram; looking for a pattern; guessing and checking; making assumptions; making an organized list; making a table or chart; making a simpler problem; working backwards; using logical reasoning.

Reasoning and Proving

An emphasis on reasoning helps students make sense of mathematics. Classroom instruction in mathematics should always foster critical thinking – that is, an organized, analytical, well-reasoned approach to learning mathematical concepts and processes and to solving problems.

As students investigate and make conjectures about mathematical concepts and relationships, they learn to employ *inductive reasoning*, making generalizations based on specific findings from their investigations. Students also learn to use counter-examples to disprove conjectures. Students can use *deductive reasoning* to assess the validity of conjectures and to formulate proofs.

Reflecting

Good problem solvers regularly and consciously reflect on and monitor their own thought processes. By doing so, they are able to recognize when the technique they are using is not fruitful, and to make a conscious decision to switch to a different strategy, rethink the problem, search for related content knowledge that may be helpful, and so forth. Students' problem-solving skills are enhanced when they reflect on alternative ways to perform a task even if they have successfully completed it. Reflecting on the reasonableness of an answer by considering the original question or problem is another way in which students can improve their ability to make sense of problems.

Selecting Tools and Computational Strategies

Students need to develop the ability to select the appropriate electronic tools, manipulatives, and computational strategies to perform particular mathematical tasks, to investigate mathematical ideas, and to solve problems.

Calculators, Computers, Communications Technology. Various types of technology are useful in learning and doing mathematics. Students can use calculators and computers to extend their capacity to investigate and analyse mathematical concepts and to reduce the time they might otherwise spend on purely mechanical activities.

Students can use calculators and computers to perform operations, make graphs, manipulate algebraic expressions, and organize and display data that are lengthier or more complex than those addressed in curriculum expectations suited to a paper-and-pencil approach. Students can also use calculators and computers in various ways to investigate number and graphing patterns, geometric relationships, and different representations; to simulate situations; and to extend problem solving. When students use calculators and computers in mathematics, they need to know when it is appropriate to apply their mental computation, reasoning, and estimation skills to predict results and check answers.

The computer and the calculator must be seen as important problem-solving tools to be used for many purposes. Computers and calculators are tools of mathematicians, and students should be given opportunities to select and use the particular applications that may be helpful to them as they search for their own solutions to problems.

Students may not be familiar with the use of some of the technologies suggested in the curriculum. When this is the case, it is important that teachers introduce their use in ways that build students' confidence and contribute to their understanding of the concepts being investigated. Students also need to understand the situations in which the new technology would be an appropriate choice of tool. Students' use of the tools should not be laborious or restricted to inputting and learning algorithmic steps. For example, when using spreadsheets and statistical software (e.g., Fathom), teachers could supply students with prepared data sets, and when using dynamic geometry software (e.g., The Geometer's Sketchpad), they could use pre-made sketches so that students' work with the software would be focused on manipulation of the data or the sketch, not on the inputting of data or the designing of the sketch.

Computer programs can help students to collect, organize, and sort the data they gather, and to write, edit, and present reports on their findings. Whenever appropriate, students should be encouraged to select and use the communications technology that would best support and communicate their learning. Students, working individually or in groups, can use computers, CD-ROM technology, and/or Internet websites to gain access to Statistics Canada, mathematics organizations, and other valuable sources of mathematical information around the world.

Manipulatives.[2] Students should be encouraged to select and use concrete learning tools to make models of mathematical ideas. Students need to understand that making their own models is a powerful means of building understanding and explaining their thinking to others. Using manipulatives to construct representations helps students to:

- see patterns and relationships;
- make connections between the concrete and the abstract;
- test, revise, and confirm their reasoning;
- remember how they solved a problem;
- communicate their reasoning to others.

Computational Strategies. Problem solving often requires students to select an appropriate computational strategy. They may need to apply the standard algorithm or to use technology for computation. They may also need to select strategies related to mental computation and estimation. Developing the ability to perform mental computation and to estimate is consequently an important aspect of student learning in mathematics.

Mental computation involves calculations done in the mind, with little or no use of paper and pencil. Students who have developed the ability to calculate mentally can select from and use a variety of procedures that take advantage of their knowledge and understanding of numbers, the operations, and their properties. Using their knowledge of the distributive property, for example, students can mentally compute 70% of 22 by first considering 70% of 20 and then adding 70% of 2. Used effectively, mental computation can encourage students to think more deeply about numbers and number relationships.

Knowing how to estimate, and knowing when it is useful to estimate and when it is necessary to have an exact answer, are important mathematical skills. Estimation is a useful tool for judging the reasonableness of a solution and for guiding students in their use of calculators. The ability to estimate depends on a well-developed sense of number and an understanding of place value. It can be a complex skill that requires decomposing numbers, compensating for errors, and perhaps even restructuring the problem. Estimation should not be taught as an isolated skill or a set of isolated rules and techniques. Knowing about calculations that are easy to perform and developing fluency in performing basic operations contribute to successful estimation.

Connecting

Experiences that allow students to make connections – to see, for example, how concepts and skills from one strand of mathematics are related to those from another – will help them to grasp general mathematical principles. As they continue to make such connections, students begin to see that mathematics is more than a series of isolated skills and concepts and that they can use their learning in one area of mathematics to understand another. Seeing the relationships among procedures and concepts also helps deepen students' mathematical understanding.

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2. See the Teaching Approaches section, on page 23 of this document, for additional information about the use of manipulatives in mathematics instruction.

In addition, making connections between the mathematics they study and its applications in their everyday lives helps students see the usefulness and relevance of mathematics beyond the classroom.

Representing

In secondary school mathematics, representing mathematical ideas and modelling situations generally takes the form of numeric, geometric, graphical, algebraic, pictorial, and concrete representation, as well as representation using dynamic software. Students should be able to go from one representation to another, recognize the connections between representations, and use the different representations appropriately and as needed to solve problems. Learning the various forms of representation helps students to understand mathematical concepts and relationships; communicate their thinking, arguments, and understandings; recognize connections among related mathematical concepts; and use mathematics to model and interpret mathematical, physical, and social phenomena. When students are able to represent concepts in various ways, they develop flexibility in their thinking about those concepts. They are not inclined to perceive any single representation as "the math"; rather, they understand that it is just one of many representations that help them understand a concept.

Communicating

Communication is the process of expressing mathematical ideas and understandings orally, visually, and in writing, using numbers, symbols, pictures, graphs, diagrams, and words. Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, or the whole class. Communication is an essential process in learning mathematics. Through communication, students are able to reflect upon and to clarify ideas, relationships, and mathematical arguments.

The development of mathematical language and symbolism fosters students' communication skills. Teachers need to be aware of the various opportunities that exist in the classroom for helping students to communicate. For example, teachers can:

- model proper use of symbols, vocabulary, and notations in oral and written form;
- expect correct use of mathematical symbols and conventions in student work;
- ensure that students are exposed to and use new mathematical vocabulary as it is introduced (e.g., by means of a word wall; by providing opportunities to read, question, and discuss);
- provide feedback to students on their use of terminology and conventions;
- ask clarifying and extending questions and encourage students to ask themselves similar kinds of questions;
- ask students open-ended questions relating to specific topics or information;
- model ways in which various kinds of questions can be answered.

Effective classroom communication requires a supportive and respectful environment that makes all members of the class comfortable when they speak and when they question, react to, and elaborate on the statements of their classmates and the teacher.

The ability to provide effective explanations, and the understanding and application of correct mathematical notation in the development and presentation of mathematical ideas and solutions, are key aspects of effective communication in mathematics.